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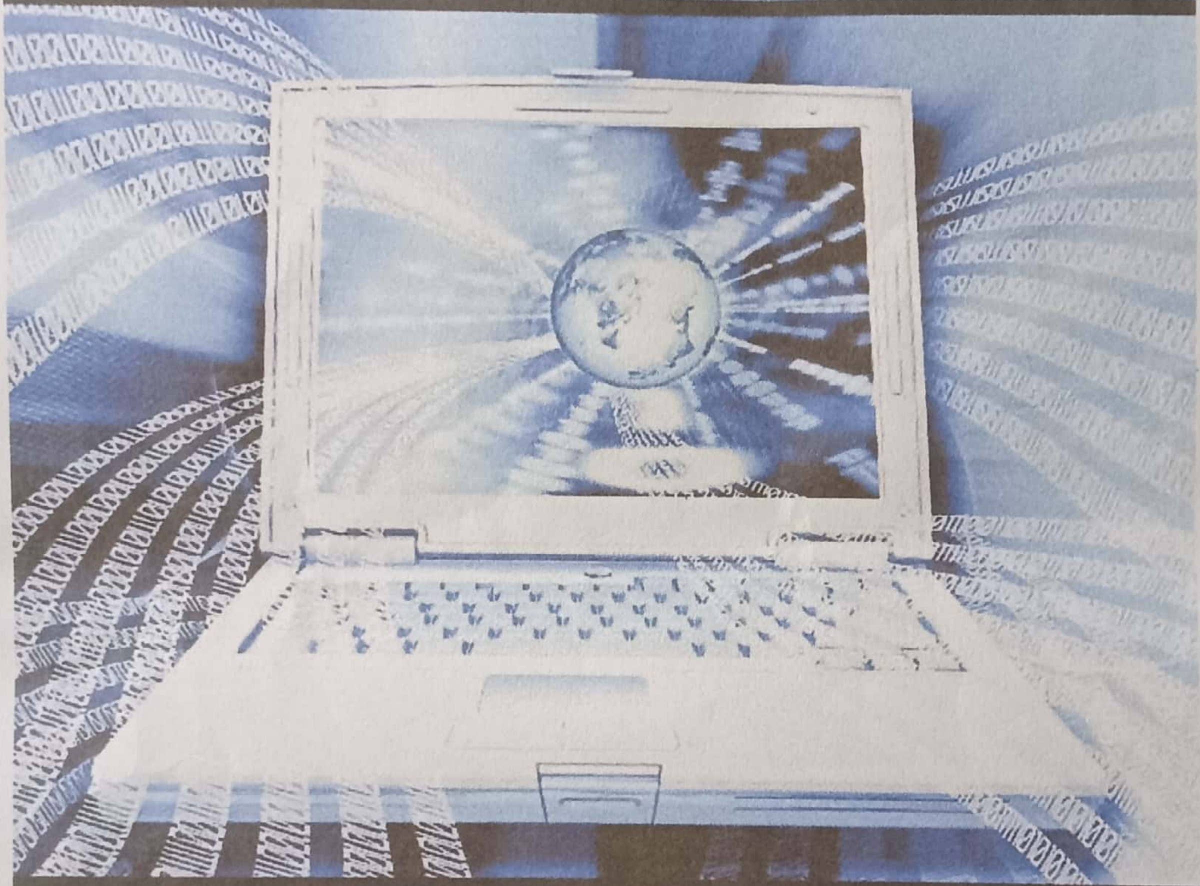

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Vol. 4

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Contents

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Even Path Decomposition of Cyclic Snake Graphs

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Abstract

An Even Decomposition (*G*2, *G*4, *G*6, …, *G*2*n*) of *G* is said to be an Even Path Decomposition (EPD) if each *G*2*i* is a path of size 2*i* and it is denoted by (*P*2,*P*4, *P*6, …, *P*2*n*). In this chapter, we introduced the concept Even Path Decomposition of Cyclic Snake Graphs.

Keywords: *Even path decomposition; odd cyclic snake graphs; triangular snake graph; even cyclic snake graphs; diamond snake graph; base path.*

1 Introduction

The origin of the study of Graph Decomposition can be seen in various combinatorial problems, which emerged in the 19th century. Graph Decomposition rank among one of the most prominent areas of Graph Theory. Many decomposition techniques were studied in literature such as [1,2] and [3] by imposing conditions on the subgraphs. In [4], double dimer covers on snake graphs from super cluster expansions were presented. All basic terminologies from Graph Theory used in this chapter in the sense of Bondy & Murty [5] and Frank Harary [6].

In this chapter, we have considered only simple and undirected graph. Let $G=(V,E)$ be the (p,q) graph, then p is called the order of G and q is called the size of G. Even path decomposition of some Even Cyclic Snake graphs is discussed in [7]. Throughout this chapter, P_{2i} denotes the path of size 2i. A cycle of length t is denoted as C_t.

The following are some of the basic definitions needed for subsequent sections.

Definition 1.1: Let $G = (V,E)$ be a simple connected graph with p vertices and q edges. If $G_1,G_2,\ldots,G_n$ are connected edge-disjoint subgraphs of G with $E(G) = E(G_1)\cup E(G_2)\cup \ldots\ldots \cup E(G_n)$, then $(G_1,G_2,\ldots,G_n)$ is said to be a **Decomposition** of G.

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Definition 1.2: A decomposition $(G_1, G_2, G_3, \cdots, G_n)$ of G is said to be an **Arithmetic Decomposition** if $|E(G_i)| = a + (i-1)d$ for all $i = 1, 2, \cdots, n$ and $a, d \in Z^+$. Clearly $q = \frac{n}{2}[2a + (n-1)d]$. If $a = 2$ and $d = 2$, then $q = n(n+1)$. That is, the number of edges of G is the sum of first n even numbers $2, 4, 6, \cdots, 2n$. Thus we call this decomposition as an **Even Decomposition**. Since the number of edges of each subgraphs of G is even, we denote the Even Decomposition as $(G_2, G_4, \cdots, G_{2n})$.

Definition 1.3: [8] A **kC_t– Cyclic Snake Graph** has been defined as a connected graph in which all the blocks are isomorphic to the cycle C_t and the block – cut point graph is a path P, where P is the path of minimum length that contains all the cut vertices of a kC_t - Cyclic Snake Graph. A kC_t - Cyclic Snake Graph has $(t-1)k+1$ vertices and tk edges where k is the number of blocks in the Cyclic Snake Graph.

Example 1.4: Throughout this chapter. the path $v_0w_1v_1w_2v_2\ldots w_kv_k$ of minimum length that contains all the cut vertices of kC_t – cyclic snake graph is considered as the base path. Here $d(v_i, v_{i+1}) = 2$, $0 \leq i \leq k-1$.

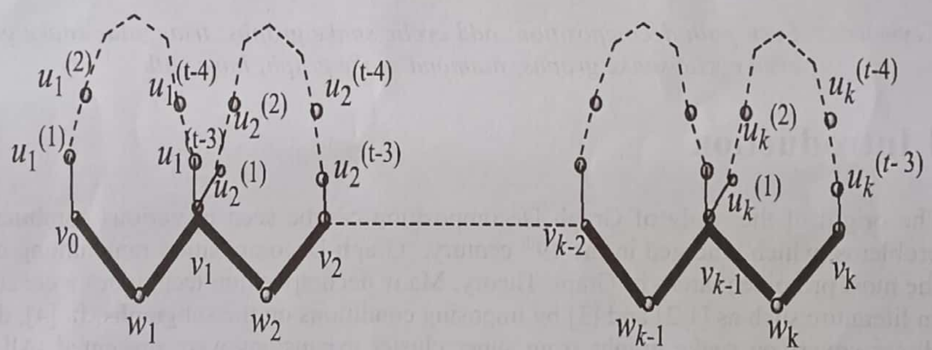

Fig. 1. k$C_t$cyclic snake graph

Definition 1.5: The **distance *d(u,v)*** between two points u and v in G is the length of the shortest path joining them.

In this chapter, we discuss about the necessary and sufficient conditions for the existence of Even Path Decomposition (EPD) in Odd Cyclic Snake graphs and Even Cyclic Snake Graphs. In particular, Even Path Decomposition of Triangular Snake graphs and Diamond Snake graphs are also discussed.

2 Even Path Decomposition of Graphs

Definition 2.1: An Even Decomposition $(G_2, G_4, G_6, \ldots, G_{2n})$ of G is said to be an **Even Path Decomposition** (EPD) if each G_{2i} is a path of size $2i$ and it is denoted by $(P_2, P_4, P_6, \ldots, P_{2n})$.

Example 2.2:

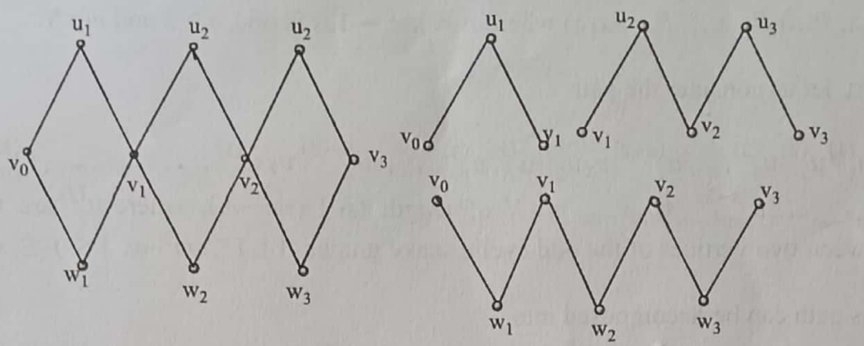

Fig. 2. 3C_4 with Even Path Decomposition (P_2, P_4, P_6)

3 Even Path Decomposition of Odd Cyclic Snake Graphs

Theorem 3.1: Odd Cyclic Snake Graphs $(sm^2-m)C_s$ admit EPD ($P_2, P_4, P_6,\ldots, P_{2n}$) if and only if $n = sm-1$, s is odd, $s \geq 3$, $m \in N$.

Proof: Assume $(sm^2-m)C_s$ admits EPD ($P_2, P_4, P_6,\ldots, P_{2n}$).

Then $q((sm^2-m)C_s) = s(sm^2-m)$. That is $s(sm^2-m) = n(n+1)$. This implies $n = sm-1$, $m \in N$.

Conversely, to prove $(sm^2-m)C_s$ admits EPD ($P_2, P_4, P_6,\ldots, P_{2n}$), where $n = sm-1$, s is odd, $s \geq 3$, $m \in N$. The proof is by induction on 'm'. The result is obvious when $m = 1$.

Suppose the result is true when $m = k$. $(sk^2-k)C_s$ admits EPD where $n = sk-1$, s is odd, $s \geq 3$.

To prove the result is true for $m = k+1$ (That is to prove $(s(k+1)^2-(k+1))C_s$ admits EPD). $(s(k+1)^2-(k+1))C_s$ can be decomposed into $(sk^2-k)C_s$ and (P_{2sk}, P_{2sk+2}, $P_{2sk+4},\ldots, P_{2(sk+s-1)}$).

By induction hypothesis $(sk^2-k)C_s$ admits EPD ($P_2, P_4, P_6,\ldots, P_{2n}$), where $n = sk-1$. Now $E((s(k+1)^2-(k+1))C_s) = E((sk^2-k)C_s) + E$ (P_{2sk}, P_{2sk+2}, $P_{2sk+4},\ldots, P_{2(sk+s-1)}$).

That is equal to $(2k+1)s^2-s$ and (P_{2sk}, P_{2sk+2}, $P_{2sk+4},\ldots, P_{2(sk+s-1)}$) are all even paths.

We conclude by the induction principle that Odd Cyclic Snake Graphs $(sm^2-m)C_s$ admits EPD ($P_2, P_4, P_6,\ldots, P_{2n}$). ■

Remark 3.2: The construction of the subgraphs $P_2, P_4, P_6,\ldots, P_{2n}$ of $(sm^2-m)C_s$ are as follows.

Let V=$\{v_0, v_1, v_2, \ldots, v_{sm^2-m}\}$,$m \in N$ be the set of vertices of the base path of $(sm^2 - m)C_s$.Let the base path be $P_{(sm^2-m)}$. $P_{(sm^2-m)}$ can be decomposed into $(P_{s-1}, P_{3s-1}, P_{5s-1}, \ldots, P_{s(2m-1)-1})$ where $n = sm - 1$, s is odd, $s \geq 3$ and $m \in N$.

Next, let us consider the path

$v_0 u_1^{(1)} u_1^{(2)} u_1^{(3)}, \ldots, u_1^{(s-2)} v_1 u_2^{(1)} u_2^{(2)} u_2^{(3)}, \ldots, u_2^{(s-2)} v_2 u_3^{(1)}, \ldots, v_{sm^2-m-1} u_{sm^2-m}^{(1)}, \ldots, u_{sm^2-m}^{(2)} \ldots u_{sm^2-m}^{(s-2)} v_{sm^2-m}$, $m \in N$ of length $(s-1)(sm^2-m)$, where $u_i^{(j)}$ are the vertices between two vertices of the odd cyclic snake graphs, $1 \leq i \leq sm^2-m$, $1 \leq j \leq s-2$.

This path can be decomposed into

$$(P_{2(s-1)}, P_{2(2s-1)}, P_{2(3s-1)}, P_{2(4s-1)}, \ldots, P_{2(ms-1)}), (P_{2(s-2)}, P_{2(2s-2)}, P_{2(3s-2)}, P_{2(4s-2)}, \ldots, P_{2(ms-2)})$$
$$(P_{2(s-3)}, P_{2(2s-3)}, P_{2(3s-3)}, P_{2(4s-3)}, \ldots, P_{2(ms-3)}), \ldots$$
$$(P_{s+1}, P_{3s+1}, P_{5s+1}, P_{7s+1}, \ldots, P_{(2m-3)s+1}, P_{(2m-1)s+1}), (P_{s-3}, P_{3s-3}, P_{5s-3}, P_{7s-3}, \ldots, P_{(2m-3)s-3}, P_{(2m-1)s-3}), \ldots,$$
$$(P_2, P_{2s+2}, P_{4s+2}, \ldots, P_{2s(m-1)+2}), (P_{2s}, P_{4s}, P_{6s}, \ldots, P_{2s(m-1)})$$

Thus we get the EPD($P_2, P_4, P_6, \ldots, P_{2n}$) of $(sm^2 - m)C_s$where $n = sm - 1$, s is odd, $s \geq 3$ and $m \in N$.

Theorem 3.3: Odd Cyclic Snake Graphs $(sm^2+ m)C_s$ admit EPD ($P_2, P_4, P_6, \ldots, P_{2n}$) if and only if $n = sm$, s is odd, $s \geq 3$, $m \in N$.

Proof: Assume $(sm^2 + m)C_s$ admits EPD ($P_2, P_4, P_6, \ldots, P_{2n}$).

Then $q((sm^2+ m)C_s) = s(sm^2+ m)$. That is $s(sm^2+ m) = n(n+1)$. This implies $n = sm$, $m \in N$.

Conversely, to prove $(sm^2 + m)C_s$ admits EPD ($P_2, P_4, P_6, \ldots, P_{2n}$), where $n = sm$, s is odd, $s \geq 3$, $m \in N$. The proof is by induction on 'm'. The result is obvious when $m = 1$.

Suppose the result is true when $m = k$. $(sk^2 + k)C_s$ admits EPD where $n = sk$, s is odd, $s \geq 3$.

To prove the result is true for $m = k + 1$ (that is to prove $(s(k+1)^2+(k+1))C_s$ admits EPD). $(s(k+1)^2+ (k+1))C_s$ can be decomposed into $(sk^2 + k)C_s$ and $(P_{2sk+2}, P_{2sk+4}, \ldots, P_{2(sk+s)})$.

By induction hypothesis $(sk^2 + k)C_s$ admits EPD ($P_2, P_4, P_6, \ldots, P_{2n}$), where $n = sk$ .Now $E((s(k+1)^2+ (k+1))C_s) = E((sk^2 + k)C_s) + E(P_{2sk+2}, P_{2sk+4}, \ldots, P_{2(sk+s)})$.

That is equal to $(2k+1)s^2+s$ and $(P_{2sk+2}, P_{2sk+4}, \ldots, P_{2(sk+s)})$ are all even paths.

We conclude by the induction principle that Odd Cyclic Snake Graphs $(sm^2+m)C_s$ admits EPD ($P_2, P_4, P_6, \ldots, P_{2n}$). ■

Remark 3.4: The construction of the subgraphs $P_2, P_4, P_6, \ldots, P_{2n}$ of $(sm^2+m)C_s$ are as follows.

Let $V=\{v_0, v_1, v_2, \ldots, v_{sm^2+m}\}, m \in N$ be the set of vertices of the base path of $(sm^2+m)C_s$. Let the base path be $P_{(sm^2+m)}$. $P_{(sm^2+m)}$ can be decomposed into $(P_{s+1}, P_{3s+1}, P_{5s+1}, \ldots, P_{s(2m-1)+1})$, s is odd, $s \geq 3$ and $m \in N$.

Next let us consider the path

$v_0 u_1^{(1)} u_1^{(2)} u_1^{(3)}, \ldots, u_1^{(s-2)} v_1 u_2^{(1)} u_2^{(2)} u_2^{(3)}, \ldots, u_2^{(s-2)} v_2 u_3^{(1)}, \ldots, v_{sm^2+m-1} u_{sm^2+m}^{(1)}, \ldots, u_{sm^2+m}^{(2)} \ldots u_{sm^2+m}^{(s-2)} v_{sm^2+m}$, $m \in N$ of length $(s-1)(sm^2+m)$.

This path can be decomposed into

$$(P_{2s}, P_{4s}, P_{6s}, P_{8s}, \ldots, P_{2ms}), (p_{2s-2}, p_{4s-2}, p_{6s-2}, p_{8s-2}, \ldots, p_{2ms-2}), \ldots, (P_{s+3}, P_{3s+3}, P_{5s+3}, \ldots, P_{(2m-1)s+3})$$
$$(P_{s-1}, P_{3s-1}, P_{5s-1}, \ldots, P_{(2m-3)s-1}, P_{(2m-1)s-1}), \ldots, (P_2, P_{2s+2}, P_{4s+2}, \ldots, P_{2s(m-1)+2})$$

Thus we get the EPD $(P_2, P_4, P_6, \ldots, P_{2n})$ of $(sm^2+m)C_s$ where $n = sm$, s is odd, $s \geq 3$ and $m \in N$.

***Definition* 3.5:** A Triangular Snake Graph $kC3$ is obtained from a path $v_0, v_1, v_2, \ldots, v_k$ by joining v_i and v_{i+1} to a new vertex u_{i+1} for $0 \leq i \leq k-1$. That is every edge of a path of size k is replaced by a triangle C_3. A Triangular Snake graph has $2k+1$ vertices and $3k$ edges where k is the number of blocks in the triangular snake graph.

***Theorem* 3.6:** Any Triangular Snake Graph $(3m^2-m)C_3$ admits EPD $(P_2, P_4, \ldots, P_{2n})$ if and only if $n = 3m-1$, $m \in$ N .

***Proof*:** Assume that $(3m^2-m)C_3$ admits EPD ($P_2, P_4, \ldots, P_{2n}$). Clearly $q((3m^2-m)C_3) = 3(3m^2-m)$. Thus $3(3m^2-m) = n(n+1)$. This implies $n = 3m-1$, $m \in$ N.

Conversely, suppose $n = 3m-1$, $m \in$ N. Let V = $\{v_0, v_1, v_2, \ldots, v_{3m^2-m}\}$, $m \in$ N be the set of vertices of the base path of $(3m^2-m)C_3$. Let the base path be P_{3m^2-m}.

Clearly, P_{3m^2-m} can be decomposed into $(P_2, P_8, P_{14}, \ldots, P_{(2m-3)3-1}, P_{(2m-1)3-1})$, $m \in$ N.

Next we consider the path $v_0\, u_1\, v_1\, u_2\, v_2\, u_3\, v_3 \ldots v_{i-1}\, u_i v_i \ldots u_{3m^2-m}\, v_{3m^2-m}$ of length $2(3m^2-m)$.

This path can be decomposed into $(P_4, P_{10}, P_{16}, \ldots, P_{2(3m-1)})$ and $(P_6, P_{12}, P_{18}, \ldots, P_{6(m-1)})$. Thus EPD of $(3m^2-m)C_3$ is $(P_2, P_4, \ldots, P_{2n})$, where $n = 3m-1$, $m \in$ N .

4 Even Path Decomposition of Even Cyclic Snake Graphs

Theorem 4.1: Even Cyclic Snake Graphs $(tm^2-m)C_t$ admit EPD $(P_2, P_4, P_6, \ldots, P_{2n})$ if and only if $n = tm-1$, t is even, $t \geq 4$, $m \in N$.

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Proof: Assume $(tm^2-m)C_t$ admits EPD ($P_2, P_4, P_6,..., P_{2n}$).

Then $q((tm^2-m)C_t) = t(tm^2-m)$. That is $t(tm^2-m) = n(n+1)$. This implies $n = tm-1,$ $m \in N$.

Conversely, to prove $(tm^2-m)C_t$ admits EPD ($P_2, P_4, P_6,..., P_{2n}$), where $n = tm-1,$ t is even, $t \geq 4$, $m \in N$. The proof is by induction on 'm'. The result is obvious when $m = 1$.

Suppose the result is true when $m = k$. $(tk^2-k)C_t$ admits EPD where $n = tk-1$, t is even, $t \geq 4$, $m \in N$.

To prove the result is true for $m = k+1$ (that is to prove $(t(k+1)^2-(k+1))C_t$ admits EPD). $(t(k+1)^2-(k+1))C_t$ can be decomposed into $(tk^2-k)C_t$ and ($P_{2tk}, P_{2tk+2}, P_{2tk+4},..., P_{2(tk+t-1)}$).

By induction hypothesis $(tk^2-k)C_t$ admits EPD($P_2, P_4, P_6,..., P_{2n}$), where $n = tk-1$. Now $E((t(k+1)^2-(k+1))C_t) = E((tk^2-k)C_t) + E(P_{2tk}, P_{2tk+2}, P_{2tk+4},..., P_{2(tk+t-1)})$.

That is equal to $(2k+1)t^2-t$ and ($P_{2tk}, P_{2tk+2}, P_{2tk+4},..., P_{2(tk+t-1)}$) are even paths.

We conclude by the induction principle that Even Cyclic Snake Graphs $(tm^2-m)C_t$ admits EPD ($P_2, P_4, P_6,..., P_{2n}$), where $n = tm-1,$ t is even, $t \geq 4$, $m \in N$.■

Remark 4.2: The construction of the subgraphs $P_2, P_4, P_6,..., P_{2n}$ of $(tm^2-m)C_t$ are as follows.

Let $\{v_0, w_1, v_1, w_2, v_2,..., w_{tm^2-m}, v_{tm^2-m}\}$, $m \in N$ be the set of vertices of the base path of $(tm^2-m)C_t$. Let the base path be $P_{2(tm^2-m)}$.

Let v_0 and $v_{(tm^2-m)}$ be the origin and terminus of the base path respectively and w_i's are the internal vertices of the base path, $0 \leq i \leq tm^2-m-1$.

$P_{2(tm^2-m)}$ can be decomposed into ($P_t, P_{3t}, P_{5t},..., P_{2mt-t}$) and ($P_{t-2}, P_{3t-2}, P_{5t-2},..., P_{t(2m-1)-2}$) where $n = tm-1,$ t is even, $t \geq 4$ and $m \in N$.

Next let us consider the path

$v_0u_1^{(1)}u_1^{(2)}u_1^{(3)},\ldots,u_1^{(t-3)}v_1u_2^{(1)}u_2^{(2)}u_2^{(3)},\ldots,u_2^{(t-3)}v_2u_3^{(1)},\ldots,v_{tm^2-m-1}u_{tm^2-m}^{(2)},\ldots,$ $u_{tm^2-m}^{(t-3)}v_{tm^2-m}$, $m\in N$ of length $(t-2)(tm^2-m)$, where $u_{i+1}^{(j)}$ are the vertices between two vertices v_i and v_{i+1} of the even cyclic snake graphs, $0\le i\le tm^2-m-1$, $1\le j\le t-3$

This path can be decomposed into $(P_{2(mt-1)}, P_{2((m-1)t-1)},\ldots, P_{2(2t-1)}, P_{2(t-1)})$, $(P_{2(mt-2)}, P_{2((m-1)t-2)},\ldots, P_{2(2t-2)}, P_{2(t-2)})$, $(P_{t+2}, P_{3t+2}, P_{5t+2},\ldots, P_{2mt-t+2}),\ldots,$ $(P_{t-4}, P_{3t-4}, P_{5t-4},\ldots, P_{2mt-t-4}),\ldots,$ $(P_{2t}, P_{4t}, P_{6t},\ldots, P_{2t(m-2)}, P_{2t(m-\square)})$.

Thus we get the EPD$(P_2, P_4, P_6,\ldots, P_{2n})$ of $(tm^2-m)C_t$ where $n=tm-1$, t is even, $t\ge 4$ and $m\in N$.

Theorem 4.3: Even Cyclic Snake Graphs $(tm^2+m)C_t$ admit EPD $(P_2, P_4, P_6,\ldots, P_{2n})$ if and only if $n=tm$, t is even, $t\ge 4$, $m\in N$.

Proof: Assume $(tm^2+m)C_t$ admits EPD $(P_2, P_4, P_6,\ldots, P_{2n})$.

Then $q((tm^2+m)C_t)=t(tm^2+m)$. That is $t(tm^2+m)=n(n+1)$. This implies $n=tm$, $m\in N$.

Conversely, to prove $(tm^2+m)C_t$ admits EPD $(P_2, P_4, P_6,\ldots, P_{2n})$ when $n=tm$, t is even, $t\ge 4$, $m\in N$. The proof is by induction on 'm'.

The result is obvious when $m=1$.

Suppose the result is true when $m=k$. $(tk^2+k)C_t$ admits EPD where $n=tk$, t is even, $t\ge 4$, $m\in N$.

To prove the result is true for $m=k+1$ (that is to prove $(t(k+1)^2+(k+1))C_t$admits EPD. $(t(k+1)^2+(k+1))C_t$ can be decomposed into $(tk^2+k)C_t$ and $(P_{2tk+2}, P_{2tk+4}, P_{2tk+6},\ldots, P_{2(tk+t)})$.

By our induction hypothesis $(tk^2+k)C_t$ admits EPD$(P_2, P_4, P_6,\ldots, P_{2n})$, where $n=tk$.

Now $E((t(k+1)^2+(k+1))C_t)=E((tk^2+k)C_t)+E(P_{2tk+2}, P_{2tk+4}, P_{2tk+6},\ldots, P_{2(tk+t)})$.

That is equal to $(2k+1)t^2+t$ and $(P_{2tk+2}, P_{2tk+4}, P_{2tk+6},\ldots, P_{2(tk+t)})$ are all even paths.

We conclude by the induction principle that Even Cyclic Snake Graphs $(tm^2+m)C_t$ admits EPD $(P_2, P_4, P_6,\ldots, P_{2n})$, where $n=tm$, t is even, $t\ge 4$, $m\in N$. ■

Remark 4.4: The construction of the subgraphs $P_2, P_4, P_6,\ldots, P_{2n}$ of $(tm^2+m)C_t$ are as follows.

Let $\{v_0, w_1, v_1, w_2, v_2,\ldots, w_{tm^2+m}, v_{tm^2+m}\}$, $m\in N$ be the set of vertices of the base path of $(tm^2+m)C_t$. Let the base path be $P_{2(tm^2+m)}$. Let v_0 and $v_{(tm^2+m)}$ be the origin and terminus of the base path respectively and w_i's are the internal vertices of the base path, $0 \leq i \leq tm^2+m-1$.

$P_{2(tm^2+m)}$ can be decomposed into (P_{t+2}, P_{3t+2}, P_{5t+2}, …, $P_{2mt-t+2}$), and (P_t, P_{3t}, P_{5t}, …, P_{2mt-t}), t is even, $t \geq 4$ and $m \in N$. Next let us consider the path, $v_0 u_1^{(1)} u_1^{(2)} u_1^{(3)}, \ldots, u_1^{(t-3)} v_1 u_2^{(1)} u_2^{(2)} u_2^{(3)}, \ldots, u_2^{(t-3)} v_2 u_3^{(1)}, \ldots, v_{tm^2+m-1} u_{tm^2+m}^{(2)}, \ldots, u_{tm^2+m}^{(t-3)} v_{tm^2+m}$, $m \in N$ of length $(t-2)(tm^2+m)$, where $u_{i+1}^{(j)}$ are the vertices between two vertices v_i and v_{i+1} of the even cyclic snake graphs, $0 \leq i \leq tm^2+m-1$, $1 \leq j \leq t-3$. This path can be decomposed into (P_{2t}, P_{4t}, P_{6t}, …, P_{2tm}), (P_{2t-2}, P_{4t-2}, P_{6t-2}, …, P_{2tm-2}),…, (P_{t+4}, P_{3t+4}, P_{5t+4}, …, $P_{2mt-t+4}$), (P_{t-2}, P_{3t-2}, P_{5t-2}, …, $P_{2mt-t-2}$), (P_{t-4}, P_{3t-4}, P_{5t-4}, …, $P_{2mt-t-4}$),…, (P_2, P_{2t+2}, P_{4t+2}, P_{6t+2}, …, $P_{2tm-2t+2}$). Thus we get the EPD (P_2, P_4, P_6,…, P_{2n}) where $n = tm$, t is even, $t \geq 4$ and $m \in N$.

Definition 4.5: A **Diamond Snake Graph** kC_4 is obtained from a path $v_0 v_1 v_2 \ldots v_k$ by joining vertices v_i and v_{i+1} to two new vertices u_{i+1} and w_{i+1} for $0 \leq i \leq k-1$. That is every edge of the path $v_0 v_1 v_2 \ldots v_k$ of size k is replaced by a cycle C_4 and $d(v_i, v_{i+1}) = 2$. A Diamond Snake graph has $3k+1$ vertices and $4k$ edges where k is the number of blocks.

Theorem 4.6: Any Diamond Snake Graph $(4m^2-m)C_4$ admits (P_2, P_4, … ,P_{2n}) if and only if $n = 4m-1, m \in N$.

Proof: Assume that $(4m^2-m)C_4$ admits EPD (P_2, P_4, … ,P_{2n}). Clearly $q((4m^2-m)C_4) = 4(4m^2-m)$. Thus $4(4m^2-m) = n(n+1)$. This implies $n = 4m-1$, $m \in N$.

Conversely suppose $n = 4m-1$, $m \in N$. Let $V = \{v_0, w_1, v_1, w_2, v_2, \ldots, w_{4m^2-m}, v_{4m^2-m}\}$, be the set of vertices of the base path of $(4m^2-m)C_4$. Let the base path be $P_{2(4m^2-m)}$. v_0 and v_{4m^2-m} are the origin and terminus of the base path respectively and w_is are the internal vertices of the base path, $1 \leq i \leq (4m^2-m)$. Clearly, $P_{2(4m^2-m)}$ can be decomposed into (P_4, P_{12}, P_{20}, …, $P_{(4m-2)2}$), and (P_2, P_{10}, P_{18}, …, $P_{2(4m-3)}$), $m \in N$.

Next we consider the path $v_0 u_1 v_1 u_2 v_2, \ldots, v_{4m^2-m-1} u_{4m^2-m} v_{4m^2-m}$ of length $2(4m^2-m)$.

This path can be decomposed into (P_6, P_{14}, P_{22}, …, $P_{2(4m-1)}$), and (P_8, P_{16}, P_{24}, …, $P_{2(4m-1)-6}$). Thus EPD of $(4m^2-m)C_4$ is (P_2, P_4, … ,P_{2n}), where $n = 4m-1$, $m \in N$. ■

Theorem 4.7: Any Diamond Snake Graph $(4m^2+m)C_4$ admits (P_2, P_4, … ,P_{2n}) if and only if $n = 4m$, $m \in N$.

Proof: Assume that $(4m^2+m)C_4$ admits EPD (P_2, P_4, … ,P_{2n}).

Clearly $q((4m^2+m)C_4) = 4(4m^2+m)$. Thus $4(4m^2+m) = n(n+1)$.This implies $n = 4m$, $m \in N$.

Conversely suppose $n = 4m$, $m \in N$.

Let $V = \{v_0, w_1, v_1, w_2, v_2, \ldots, w_{4m^2+m}, v_{4m^2+m}\}$,$m \in N$ be the set of vertices of the base path of $(4m^2+m)C_4$. Let the base path be $P_{2(4m^2+m)}$. v_0 and v_{4m^2+m} are the origin and terminus vertices of the base path respectively and w_is are the internal vertices of the base path, $1 \le i \le (4m^2+m)$. Clearly, $P_{2(4m^2+m)}$ can be decomposed into $(P_6, P_{14}, P_{22}, \ldots, P_{2(4m-1)})$ and $(P_4, P_{12}, \ldots, P_{2(4m-2)})$, $m \in N$. Next we consider the path $v_0 u_1 v_1 u_2 v_2, \ldots, v_{4m^2+m} u_{4m^2+m} v_{4m^2+m}$ of length $2(4m^2+m)$. This path can be decomposed into $(P_8, P_{16}, P_{24}, \ldots, P_{2(4m)})$ and $(P_2, P_{10}, \ldots, P_{2(4m-3)})$.Thus EPD of $(4m^2+m)C_4$ is $(P_2, P_4, \ldots, P_{2n})$, where $n = 4m$, $m \in N$. ■

5 Conclusion

In this chapter, Even Path Decomposition of Cyclic Snake Graphs has been discussed.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Ebin Raja Merly E, Gnanadhas N. Linear path decomposition of lobster. International Journal of Mathematics Research. 2011;3(5):447-455.

[2] Ebin Raja Merly E, Jaya Jothi D. Connected domination path decomposition of triangular snake graph. International Journal of Pure and Applied Mathematics. 2017;116:105-113.

[3] Viji SP. Even path decomposition of geometric mean graphs. JETIR. 2021;8(6):575-582.

[4] Musiker G, Ovenhouse N, Zhang SW. Double dimer covers on snake graphs from super cluster expansions. Journal of Algebra. 2022;608:325-381.

[5] Bondy JA, Murthy USR. Graph Theory with Applications, Elseiver Science Publishing Co Inc; 1982.

[6] Frank Harary. Graph Theory, Addison – Wesley Publishing Company; 1969.

[7] Suthiesh Goldy J, Ebin Raja Merly E. Even Path decomposition of Even Cyclic Snake Graphs. Journal of Computer and Mathematical Sciences, 2018 June;9(6):625-630.

[8] LourduSamy A, Sherry George. Super vertex mean labeling of cyclic snakes. Information in Sciences and Computing. Volume 2016, Article ID ISC630416, 25 pages.
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